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Endogenizing Private Information: Incentive Contracts under Learning By Doing*

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Abstract

This paper investigates the design of incentives in a dynamic adverse selection framework when agents' production technologies display learning effects and agents' rate of learning is private knowledge. In a simple two-period model with full commitment available to the principal, we show that whether learning effects are over- or under-exploited crucially depends on whether learning effects increase or decrease the principal's uncertainty about agents' costs of production. Hence, what drives the over- or under-exploitation of learning effects is whether more efficient agents also learn faster (so costs diverge through learning effects) or whether it is the less efficient agents who learn faster (so costs converge). Furthermore, we show that if divergence in costs through learning effects is strong enough, learning effects will not be exploited at all, in a sense to be made precise.

JEL Classification: D82, L14, L43, L51, O31

Keywords: Asymmetric Information, Learning by Doing, Regulation

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1 Introduction

Supply relations in which a firm procures a good from another firm are typically ridden with problems of asymmetric information. The prototypical case considered in the literature is that of a supplier enjoying private information concerning his marginal cost of producing the good (see for instance Laffont and Martimort, 2002, for a textbook treatment). What is characteristic of these models is the assumption that the supplier's marginal costs are exogenously given. However, the production of many of the goods typically traded in such supplier relations involves learning effects, meaning that the supplying firm can lower its marginal costs as it gains experience with rising production volumes.[1] Such relations will therefore involve endogenous cost structures as the supplier's marginal costs (and thereby the extent to which he is privately informed) come to depend not only on exogenous factors, but also on the endogenous volume of trade. This paper explores the impact of endogenizing marginal costs on optimal procurement: Does private information concerning suppliers' extent of learning effects lead to contracts that exploit learning effects only to an inefficiently low extent? More generally, how does the endogenous formation of agents' private information affect contractual arrangements?

Such questions arise not only in traditional vertical procurement relations. More recently, Baron and Myerson's (1982) classical question of how to regulate a monopolistic service supplier with private information on costs has been revisited (see Lewis and Yildirim, 2002a). While we already have a fairly good understanding of optimal regulation in stationary settings, relatively little is still known about the impact of regulation in dynamic settings where suppliers' technology is endogenous. Specifically, technological improvements through learning effects should play a significant role in electricity or telecommunication markets, for instance, and it is important to understand the long-run interplay between regulation and innovation. Again, we may wonder whether regulation in such a setting will lead to learning effects being under-exploited and hence result in inefficiently low innovation.

To tackle these questions, we set up a simple model of procurement over two periods, where in each period, the supplier produces and sells some amount of a good to the procuring firm in exchange for a monetary transfer. First period marginal costs are exogenously given (and publicly known). However, to capture learning effects, second period marginal costs are assumed to depend on the amount of first

[1] Learning effects have been documented in numerous industries, such as in the production of airplanes (Wright, 1936; Alchian, 1963), ships (Rapping, 1965; Thompson, 2001; Thornton and Thompson, 2001), chemicals (Stobaugh and Townsend, 1975; Lieberman, 1984), machine tools (Hirsch, 1952, 1956), computers and semiconductors (Nye, 1996; Gruber, 1998), electrical equipment (Preston and Keachie, 1964; Sultan, 1975), nuclear power (Joskow and Rozanski, 1979; Roberts and Burwell, 1981; Lester and McCabe, 1993) and in the weapons industry (Fox, 1988; Gansler, 1989). While these studies all pertain to cost reducing learning effects, a recent strand of literature also seeks do document qualitative learning effects in production.(see Moul, 2001)

period production. Furthermore, the strength of this learning effect is assumed to be suppliers' private knowledge. We assume that, before the first period, the principal can offer a menu of quantities and transfers over both periods, and that he can commit not to renege on this contract.

In this setting, we derive the following set of results: First, whether learning effects are over- or under-exploited in equilibrium crucially depends on whether agents' learning technologies are such that second-period marginal costs converge or diverge with increasing first-period output: If second-period marginal costs diverge, first-period production will be inefficiently low, whereas if second-period marginal costs converge, first-period production will be inefficiently high, where the size of the distortion in either case will depend on the rate of divergence or convergence. In this context, the efficiency benchmark for learning by doing is the socially efficient level of first period output *given* the equilibrium level of second period output. Second, we argue that when comparing outcomes under private information to the unconstrained full information benchmark (i.e., without holding second period output fixed), the above categorization according to convergence or divergence of agents' second period costs no longer suffices to derive robust qualitative results. Only in the first case of second-period marginal costs diverging can we generally show both periods' outputs to fall short of their efficient levels, whereas the overall impact under converging costs remains open. Thirdly, we show that if second-period marginal costs diverge sufficiently with increasing output, learning effects may even be non-exploited in the sense of first-period output falling short of its static optimum.

The paper closest in spirit to ours is Lewis and Yildirim (2002a). In the context of optimal regulation of a service provider, the authors analyze how dynamic regulation deals with suppliers' learning effects under private information (in a companion paper, Lewis and Yildirim, 2002b, also apply these results to vertical procurement relations). One of its key conclusions is that by means of a more light-handed regulatory approach, optimal policy indeed encourages learning effects, but at an inefficiently low level. However, their model differs from ours in some important respects. First, private information in their model pertains to the *cost* side of learning effects: while agent and principal have symmetric knowledge concerning the impact of higher output today on marginal costs tomorrow (i.e. on learning effects themselves), each period involves a transitory 'cost-shock' which offsets production costs in that period only and which is known only to the agent. Hence, if we think of learning effects as inducing a cost-benefit trade-off—where costs are inefficiently high output today from a static viewpoint and benefits are lower marginal costs tomorrow—then private information in their model concerns the cost side of this trade-off. In contrast, this paper focusses on the benefit side by letting agents enjoy private information on their learning technology.[2]

[2]There are some further differences between the models: First, whereas in Lewis and Yildirim

The rest of this paper is organized as follows. Section 2 sets up a basic two-period model of learning by doing and describes the full information benchmark. Returning to the private information case, Section 3 works out the optimal contract under the presumption that the principal can fully commit and discusses both its efficiency properties and whether learning effects are even made use of in the first place. Section 4 concludes with a brief discussion of the results and their applications and discusses possible extensions.

2 A Simple Two-Period Model of Learning By Doing

2.1 Setting up the Model

Consider a simple model in which a principal procures a good from an agent over two periods. In each period, the principal's utility is given by

$$v_t = S(q_t) - z_t, \qquad t \in \{1, 2\},$$

where q_t is the amount of the good delivered to the principal in period t, and z_t is the transfer made from the principal to the agent in that period. We assume that $S' > 0$, $S'' < 0$, and $S'(0) = +\infty$, an Inada-type condition ensuring that shutdown is never optimal. The principal's discount factor is δ, so that his overall utility from transactions over the two periods is given by $V = v_1 + \delta v_2$.[3]

In each period, the agent produces the good at constant marginal cost c_t, so that his utility in period t is given by

$$u_t = z_t - c_t q_t, \qquad t \in \{1, 2\}.$$

The agent has the same discount factor as the principal, so that the agent's overall utility is given by $U = u_1 + \delta u_2$. Furthermore, to keep matters simple, both principal and agent are assumed to be risk neutral.

The agent's marginal cost in period 1, c_1, is observable and known *ex ante* to both agent and principal. The agent's private knowledge concerns the structure of

(2002a), agents' private information is independent across periods due to the transitory nature of cost shocks, our model assumes agents' private information to persist in the form of agents' inherent learning abilities. Second, Lewis and Yildirim (2002a) only let the principal offer spot-contracts every period, whereas we permit the principal to offer a long-term contract and let him commit not to renege on this contract.

[3]Note that we have assumed the principal's marginal utility of to be the same across periods (up to a discount factor), which reflects our learning by doing setting. However, by a simple extension, pure investment situations could be modelled by setting the principal's marginal utility of first period output to zero. Our model would then describe a situation in which the investment made by the supplier is observable, but the effects of the investment on future marginal costs are unknown to the principal.

costs in period 2, c_2. Marginal costs in period 2 are assumed to depend both on private knowledge held by the agent (his type $\theta \in \Theta$), and the amount of the good produced in period 1 (q_1):

$$c_2 = c_2(\theta, q_1),$$

where $c_2 > 0$, $\partial c_2/\partial q_1 < 0$, and $\partial^2 c_2/\partial q_1^2 \geq 0$. The assumption that $\partial c_2/\partial q_1 < 0$ captures the learning by doing effect in production: The more of the good the agent produces in period 1, the lower the marginal costs of further production in period 2 for any given type θ.[4] The condition on the second order derivative with respect to q_1 ensures that learning effects don't grow arbitrarily large. Note however that this simplifying assumption will need to be further strengthened to derive many of our results below.

Further, we normalize private information (i.e. types) by assuming that $\partial c_2/\partial\theta < 0$, so that a high value of θ is an indication of low marginal costs in period 2 (holding constant the level of output in period 1, q_1). Specifically, this implies that for any first period output level q_1, a higher θ-type always is more efficient in the second period.

Figure 1 gives examples of period-two marginal cost functions which are compatible with our assumptions by plotting sample cost functions for two particular types $\overline{\theta} > \underline{\theta}$. Note that at this point we have made no assumptions concerning the cross derivative $\frac{\partial^2}{\partial q_1 \partial\theta}c_2$, which describes the difference in learning rates among agents. While no assumptions are imposed on the sign itself, we shall assume however, that the sign is constant, i.e. that $\frac{\partial^2}{\partial q_1 \partial\theta}c_2$ is either positive or negative over the entire relevant range. Furthermore, as this distinction will turn out to be important, we introduce the following straightforward terminology:

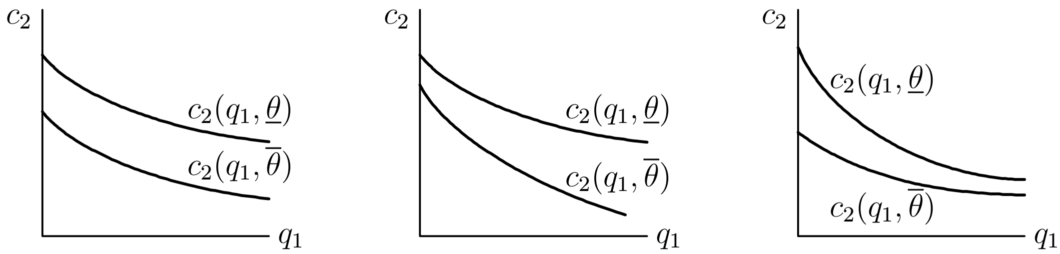
Definition 1 (Learning Rates). For two types $\theta_1, \theta_2 \in \Theta$, we shall say that the θ_1-type *learns faster* than the θ_2-type if

$$\left|\frac{\partial}{\partial q_1}c_2(q_1,\theta_1)\right| > \left|\frac{\partial}{\partial q_1}c_2(q_1,\theta_2)\right| \qquad \text{for all } q_1 \geq 0.$$

In terms of the cross-partial $\frac{\partial^2}{\partial q_1 \partial\theta}c_2$, we may thus say that (i) more efficient agents (with a higher θ) learn faster whenever $\frac{\partial^2}{\partial q_1 \partial\theta}c_2 < 0$, whereas (ii) less efficient agents (those with a lower θ) learn faster whenever $\frac{\partial^2}{\partial q_1 \partial\theta}c_2 > 0$. Finally, if $\frac{\partial^2}{\partial q_1 \partial\theta}c_2 = 0$, agents learn at the same rate. Figure 1 shows examples for the different cases.

Summing up, we thus have two qualitative dimensions along which to differentiate agents: First, the assumption that $\partial c_2/\partial\theta < 0$ lets us say that high-θ agents are 'good' (and low-θ agents 'bad') in the sense of being more efficient in the second

[4] Note we assume marginal costs to be constant *within* each period but change discontinuously from one period to the next. Indeed, what distinguishes learning effects from simple scale economies is that learning by doing depends on both previous production volumes and on time.



(a) $\partial^2 c_2/\partial q_1 \partial\theta = 0$; agents learn at the same rate.

(b) $\partial^2 c_2/\partial q_1 \partial\theta < 0$; high-$\theta$ agents learn faster.

(c) $\partial^2 c_2/\partial q_1 \partial\theta > 0$; low-$\theta$ agents learn faster.

Figure 1: Types of Learning Effects, illustrated for two Types $\overline{\theta} > \underline{\theta}$.

period given any first period output—a terminology which we shall entertain for the remainder. However, this assumption does not preclude the 'bad' agent from learning faster than the 'good' agent in the sense just discussed. These constellations are again illustrated in Figure 1.

Finally, for expositional simplicity, we shall assume for simplicity that there are only two possible types of agents, so $\theta \in \{\underline{\theta}, \overline{\theta}\} \equiv \Theta$. The agent knows his own type, whereas the principal only knows the *ex ante* distribution of types, as given by $\text{Prob}(\theta = \overline{\theta}) = \nu$.

2.2 The Full Information Benchmark

As a point of comparison, let us briefly consider the full information benchmark. Assume for a moment that the agent's type θ is publicly known. Given any type $\theta \in \Theta$, the joint surplus of the relationship is given by

$$S(q_1) - c_1 q_1 + \delta[S(q_2) - c_2(q_1, \theta) q_2].$$

The principal will then offer a contract specifying (type-dependent) production levels q_1 and q_2 so as to maximize joint surplus, so that first best levels of production q_1^*

and q_2^* for a θ-type satisfy the following first order conditions:[5]

$$S'(q_1^*) = c_1 + \delta q_2^* \frac{\partial c_2(q_1^*, \theta)}{\partial q_1} \tag{1}$$

$$S'(q_2^*) = c_2(q_1^*, \theta). \tag{2}$$

By equation (2), production in period 2 is chosen simply to equate marginal utility to the principal with marginal costs to the agent. The second term in equation (1) shows that the choice of production level in period 1 must not only take into account the direct benefits of the good to the principal as captured by $S(q_1)$, but also cost savings which higher production in period 1 implies for production in period 2 due to learning effects. Hence (recall that $\partial c_2/\partial q_1 < 0$ by assumption), production in period 1 will be *higher* than what would result if the choice of production levels would ignore learning effects.

As this will prove helpful for our derivations later on, let us define the following two functions, which describe each period's optimal output level given output in the other period: With slight abuse of notation, let $q_1^*(q_2|\theta)$ denote the function that assigns each given second-period output level q_2 the optimal first-period output level according to (1), and similarly let $q_2^*(q_1|\theta)$ denote the function that assigns a given first-period output level q_1 the optimal second-period output level according to (2), each dependent on the given type θ. The following properties of these functions are then straightforward to derive using fist-order conditions (1) and (2) and the assumed concavity of the social objective function:

Lemma 1. *The conditional first-best levels of output have the following properties (conditional on a given θ):*

1. *$q_1^*(q_2|\theta)$ is strictly increasing in q_2, and*
2. *$q_2^*(q_1|\theta)$ is strictly increasing in q_1.*

Intuitively, a higher level of second-period output raises the incentive to lower marginal second-period costs by choosing a higher level of first-period output. On the other hand, a higher level of first-period output simply has the effect of lowering second-period marginal costs, which induces higher second-period output. At a more abstract level, these results are driven by the fact that first- and second-period

[5] While the surplus $S(q_1) + \delta S(q_2)$ is clearly concave in q_1 and q_2, costs are generally nonconvex. Thus, to ensure the validity of the first-order approach (and to make sure the maximization problem is well defined in the first place), we shall assume that learning rates $|\frac{\partial}{\partial q_1} c_2|$ are not too large. Specifically, the above objective function is concave if

$$\left(S''(q_1) - \delta q_2 \frac{\partial^2}{\partial q_1^2} c_2(q_1, \theta)\right) S''(q_2) \geq \delta \left(\frac{\partial}{\partial q_1} c_2(q_1, \theta)\right)^2,$$

which will be satisfied (given $S(\cdot)$ and δ) whenever learning effects are not too large.

output are complements in the social objective function, so that raising one increases the marginal social value to rasing the other.

Finally, observe that the discounted sum of first best transfers z_1^* and z_2^* will be pinned down by the agent's participation constraint so as to leave the agent at his reservation utility. If we assume that the agent's participation constraint takes the form $U(\theta) \geq 0$, $\forall\theta$, then any pair of transfers $\{z_1^*, z_2^*\}$ such that

$$z_1^* + \delta z_2^* = c_1 q_1^* + \delta c_2(q_1^*, \theta) q_2^*$$

will be optimal from the principal's point of view. The ambiguity in the optimal monetary transfer scheme is a consequence of the assumption of identical discount factors and risk attitude.

3 The Optimal Contract under Full Commitment

Let us now reintroduce the assumption that agents' learning by doing parameter θ is private knowledge. The first best contracts described above are then generally no longer feasible, as at least one type of agent will want to claim to be of the other type in order to reap a higher rent (technically, at least one type of agent's incentive constraint will typically be violated, as seen below). Hence, the principal will need to offer a different contract, taking into account agents' possibility of lying about their type, which leads to extra restrictions on feasible contracts.

In a dynamic setting, it is important to be clear about the level of commitment available to the principal. For now, we shall assume that at the start of the first period, the principal can offer a long-term contract spanning both periods of gameplay, and that he can commit not to renege on this contract.

3.1 Characterizing the Optimal Contract

The full commitment setting has the nice property that by the revelation principle (see Fudenberg and Tirole, 1991, for instance), we may equivalently restrict our attention to truth revealing mechanisms of the type $\{z_1(\theta), z_2(\theta), q_1(\theta), q_2(\theta)\}_{\theta\in\Theta}$, which specify transfers and the traded amounts of the good as a function of the type and where each agent truthfully reveals his type.[6] In our two-type model, such a mechanism will consist of a menu of two contracts. To allow for more compact notation, we will often drop the argument θ in favor of an upper or a lower bar in

[6]For practical purposes, it is important to bear in mind that the revelation principle merely yields an equivalent representation of the optimal mechanism, whereas the actual implementation used may look quite different (for instance, non-linear price schedules may be used instead of type dependent menus).

what follows, so that

$$(\overline{z_1}, \overline{z_2}, \overline{q_1}, \overline{q_2}) \equiv [z_1(\overline{\theta}), z_2(\overline{\theta}), q_1(\overline{\theta}), q_2(\overline{\theta})],$$
$$(\underline{z_1}, \underline{z_2}, \underline{q_1}, \underline{q_2}) \equiv [z_1(\underline{\theta}), z_2(\underline{\theta}), q_1(\underline{\theta}), q_2(\underline{\theta})].$$

In order to describe incentive compatible contracts more elegantly (see Laffont and Martimort, 2002), define the θ-type agent's overall utility $U(\theta)$ resulting from 'his' contract $\{z_1(\theta), z_2(\theta), q_1(\theta), q_2(\theta)\}$ as

$$U(\theta) = z_1(\theta) - c_1 q_1(\theta) + \delta[z_2(\theta) - c_2(q_1(\theta), \theta) q_2]$$

for all $\theta \in \Theta$. Then, rather than describing contracts as a menu of type dependent payments and production schedules, we may describe them as a menu of *rent* and production schedules, $\{U(\theta), q_1(\theta), q_2(\theta)\}_{\theta\in\Theta}$ in a payoff equivalent way.[7] (Again, for notational simplicity in the two-type case, we will frequently use $\overline{U} \equiv U(\overline{\theta})$ and $\underline{U} \equiv U(\underline{\theta})$ below.)

Next, let us introduce the following useful function which describes the *utility differential* between types for a given contract:

$$\Phi(q_1, q_2) \equiv \delta q_2[c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})], \tag{3}$$

Specifically, for a given contract (specifying output schedule (q_1, q_2) and transfer schedule (z_1, z_2)), $\Phi(q_1, q_2)$ gives the difference in utility between an efficient $\overline{\theta}$-type and an inefficient $\underline{\theta}$-type due to cost differences between types under this contract.[8]

Note some straightforward technical properties of the $\Phi(q_1, q_2)$-function over the relevant range, which will be useful later on (see Appendix A.1 for the derivations):

Lemma 2. *The utility differential $\Phi(q_1, q_2)$ has the following properties:*

1. $\Phi(q_1, q_2) \geq 0$, *with the inequality being strict whenever* $q_2 > 0$,
2. $\partial\Phi/\partial q_2 > 0$,
3. $\mathrm{sgn}[\partial\Phi/\partial q_1] = -\mathrm{sgn}[\partial^2 c_2/(\partial q_1 \partial\theta)] \cdot \mathrm{sgn}[q_2]$.
4. $\mathrm{sgn}[\partial^2\Phi/\partial q_1 \partial q_2] = -\mathrm{sgn}[\partial^2 c_2/(\partial q_1 \partial\theta)] \cdot \mathrm{sgn}[q_2]$
5. $\Phi(q_1, q_2)$ *is convex in* (q_1, q_2) *if and only if* $\partial^2 c_2/(\partial q_1 \partial\theta) = 0$.

[7]'Payoff equivalent' means that such contracts of course only specify aggregate discounted transfers $z_1(\theta) + \delta z_2(\theta)$, but not the exact split of transfers across periods. To achieve the latter, we would need to consider contracts of the type $\{u_1(\theta), u_2(\theta), q_1(\theta), q_2(\theta)\}_{\theta\in\Theta}$, where u_t is the agent's utility levels in period t. This presents no restriction in the current setting however, as both the principal's and the agent's payoffs are affected only by aggregate discounted transfers.

[8]Since transfers (z_1, z_2) enter utility in the same additive fashion for both types, they do not affect the utility differential, which is why $\Phi(\cdot)$ depends only on the arguments q_1 and q_2.

The first assertion is almost tautological by noting that the more efficient agent's rent will exceed the less efficient agent's rent for any given contract specifying a given pair of transfers and quantities q_1 and q_2. The second assertion states that the utility differential is strictly increasing in period-two output, as the lower second-period marginal costs of the efficient $\overline{\theta}$-type are magnified by a larger output. Assertion 3 notes that the utility differential is increasing in q_1 if the more efficient agent also learns faster, and decreasing in q_1 if the inefficient agent learns faster. For a strictly positive, fixed level of second-period output, the utility differential is of course driven by types' difference in marginal costs. While both are decreasing in q_1 by assumption in absolute terms, the utility differential is driven by types' relative learning rates. As Figure 1 illustrates, second-period costs diverge (i.e. the difference increases) if the more efficient type also learns faster, whereas they converge if the inefficient type learns faster. This drives the sign on $\frac{\partial}{\partial q_1}\Phi$. This impact of first-period output on the utility differential is of course magnified by a larger volume of second-period output for the marginal cost-differences to materialize over, which is why $\frac{\partial}{\partial q_1}\Phi$ and $\frac{\partial^2}{\partial q_1 \partial q_2}\Phi$ have the same sign, as noted in assertion 4. Finally, assertion 5 notes that in all cases but the degenerate one in which types learn at the same rate, the utility differential is not convex in its arguments, which will be important further down.

For now, we shall further assume that reservation utilities are not type dependent and normalized to zero, so that the participation constraints take the form

$$U(\overline{\theta}) \geq 0 \qquad (\overline{\text{PC}})$$
$$U(\underline{\theta}) \geq 0. \qquad (\underline{\text{PC}})$$

This may be interpreted as both types facing identical outside options, i.e. the good type not having better prospects outside of the relationship with the principal than the bad type.

Given these preliminaries, we may state the following proposition, which permits us to greatly simplify the description of the principal's mechanism design problem:

Proposition 1. (a) *The uninformed principal's problem of designing the optimal incentive compatible contract can equivalently be expressed as choosing type-dependent allocations, $(\overline{q_1}, \overline{q_2})$ and $(\underline{q_1}, \underline{q_2})$, so as to maximize his expected payoff,*

$$\begin{aligned}\Pi(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_2}) \equiv \quad & \nu\big\{S(\overline{q_1}) - c_1\overline{q_1} + \delta[S(\overline{q_2}) - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\big\} \\ & + (1-\nu)\big\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\big\} \\ & - \nu\Phi(\underline{q_1}, \underline{q_2}),\end{aligned} \tag{4}$$

subject to the implementability condition

$$\Phi(\overline{q_1}, \overline{q_2}) \geq \Phi(\underline{q_1}, \underline{q_2}), \tag{5}$$

and settings types' rents (and thereby implicitly transfers) at $\overline{U} = \Phi(\underline{q_1}, \underline{q_2})$ *and* $\underline{U} = 0$.

(b) *Furthermore, whenever the principal's objective function in (4) is concave in its arguments, the implementability condition (5) will not be binding at the optimum.*

The proofs are given in the Appendix. Proving part (a) involves explicitly spelling out the types' incentive constraints using the notation developed above, and then employing standard techniques to derive the implementability condition and to show that only the efficient types' incentive constraint and the inefficient types' participation constraint will be binding at the optimum. Proving part (b) is somewhat less standard and more involved.

The reduced form of the principal's payoff as given in (4) can be explained by noting that the first two lines of (4), to which we shall assign the function $W(\cdot)$ as

$$\begin{aligned} W(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_2}) \equiv \quad & \nu\{S(\overline{q_1}) - c_1\overline{q_1} + \delta[S(\overline{q_2}) - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\} \\ & + (1-\nu)\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\}, \end{aligned} \tag{6}$$

correspond to the expected social surplus of the allocation $(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_2})$. Indeed, the socially optimal allocation as described in the previous section is obtained by maximizing $W(\cdot)$ with respect to its arguments. Clearly, this would also be the principal's *ex ante* expected payoff if he were able to verify the agent's type and thus appropriate the full surplus from dealing with either possible type of agent. As this is not the case under incomplete information, however, the principal must take into account the two types' incentive constraints in order to keep agents from misreporting their types. As one might expect, the relevant incentive problem turns out to be the more efficient agent claiming to be of the inefficient type in order to obtain a higher compensation for his allegedly higher costs. Thus, the efficient $\overline{\theta}$-type agent must be given an informational rent equal to the utility he could gain by claiming to be a $\underline{\theta}$-type, which by the definition of the Φ-function as the utility differential between types is given by $\Phi(\underline{q_1}, \underline{q_2})$. As this informational rent needs to be paid with probability ν (the probability of dealing with an efficient type), incomplete information causes the principal's expected payoff to be diminished by $\nu\Phi(\underline{q_1}, \underline{q_2})$, so that his expected utility is given by

$$\Pi(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_2}) = W(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_2}) - \nu\Phi(\underline{q_1}, \underline{q_2}),$$

as shown in (4).

Finally, let us shed some light on the meaning of the implementability condition (5). Recalling the interpretation of Φ as a utility differential between types for a given contract, condition (5) says that the gains from being a good rather than a bad type must be larger under the contract designed for the good type (involving production volumes $\overline{q_1}$ and $\overline{q_2}$) than under the contract designed for the bad type (involving production volumes $\underline{q_1}$ and $\underline{q_2}$). Equivalently, the $(\overline{q_1}, \overline{q_2})$-allocation must involve larger cost savings of the good over the bad type than the $(\underline{q_1}, \underline{q_2})$-allocation. Recall from our previous discussion that any incentive compatible contract must involve the efficient $\overline{\theta}$-type getting a rent of at least $\Phi(\underline{q_1}, \underline{q_2})$ over the inefficient $\underline{\theta}$-type's to keep him from pretending to be inefficient. At the same time, the inefficient $\underline{\theta}$-agent could always misrepresent his type which would then increase his rent by at least $\Phi(\underline{q_1}, \underline{q_2})$ less the amount by which the good type's costs exceed the bad type's for that contract, namely $\Phi(\overline{q_1}, \overline{q_2})$. Whenever the implementability condition is violated, this deviation is obviously profitable. Thus, allocations which violate the implementability condition cannot be implemented because there is no choice of transfers that could make both agents truthfully reveal their type.[9]

Part (b) of Proposition 1 shows however that, as one might expect intuitively, this restriction on incentive feasible allocations can be neglected whenever the principal's objective is concave in its arguments. In other words, this condition need not concern the principal in finding the optimal contract since a contract violating the implementability condition can never be (unconstrained) optimal in the first place. The reason why we have only shown this for concave objective functions is somewhat subtle and has to do with the fact that as soon as the principal's objective is not concave, the optimal mechanism will often be stochastic. In the present context, a stochastic mechanism would be one which, for every revealed type, the agent is asked to implement one out of a range of output combinations at random according to a prespecified probability distribution. However, such stochastic mechanisms bring with them a number of conceptual problems (see for instance Laffont and Martimort, 2002, pp. 65–67, on stochastic mechanisms in the standard model), which is why for the remainder of our analysis, we shall restrict ourselves to situations in which the principal's objective (4) is concave.[10]

[9] Note that in contrast to the standard static model, this implementability condition involves a joint restriction on *both* periods' production levels. The shape this constraint takes crucially depends on the cross derivative $\partial^2 c_2/(\partial q_1 \partial \theta)$. If $\partial^2 c_2/(\partial q_1 \partial \theta) = 0$, then $\partial \Phi/\partial q_1 = 0$ by Lemma 2, so that we retrieve the implementability condition of the static model for second period outputs only, namely $\overline{q_2} \geq \underline{q_2}$. Intuitively then, first period production volumes have no impact on cost savings of the good over the bad type, and hence are irrelevant for the separation of types.

[10] Observe that concavity of the first best objective function $W(\cdot)$, as implicitly required in our above first-best analysis, is not sufficient here to guarantee concavity of the full objective function since we have seen above in Lemma (2) that $\Phi(q_1, q_2)$ is generally non-convex. However, we may again spell out this concavity requirement explicitly. Letting $g(\underline{q_1}, \underline{q_2}) \equiv (1-\nu)\big\{S(\underline{q_1}) - c_1 \underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\big\} - \nu\Phi(\underline{q_1}, \underline{q_2})$ denote the objective function and using the second order partials of Φ given in Section A.1 in the Appendix, we have

$$\frac{\partial^2}{\partial \underline{q_1}^2} g = (1-\nu)\left[S''(\underline{q_1}) - \delta \underline{q_2} \frac{\partial^2}{\partial \underline{q_1}^2} c_2(\underline{q_1}, \underline{\theta})\right] - \nu\delta \underline{q_2}\left[\frac{\partial^2}{\partial \underline{q_1}^2} c_2(\underline{q_1}, \underline{\theta}) - \frac{\partial^2}{\partial \underline{q_1}^2} c_2(\underline{q_1}, \overline{\theta})\right]$$
$$\frac{\partial^2}{\partial \underline{q_2}^2} g = \delta(1-\nu) S''(\underline{q_2})$$
$$\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} g = -\delta(1-\nu)\frac{\partial}{\underline{q_1}} c_2(\underline{q_1}, \underline{\theta}) - \nu\delta\left[\frac{\partial}{\partial \underline{q_1}} c_2(\underline{q_1}, \underline{\theta}) - \frac{\partial}{\partial \underline{q_1}} c_2(\underline{q_1}, \overline{\theta})\right],$$

3.2 Efficiency Properties of the Optimal Contract

Having greatly simplified the characterization of the principal's optimal contract under incomplete information by means of Proposition 1, we now proceed to investigate its efficiency properties. Specifically, we shall try to derive qualitative differences between quantities specified in the contract offered by the uninformed principal and the optimal quantities under full information as derived in Section 2.

Recall to this end that the efficient levels of output, denoted $\{\underline{q_1}^*, \underline{q_2}^*, \overline{q_1}^*, \overline{q_1}^*\}$, maximize expected social welfare $W(\cdot)$:

$$(\underline{q_1}^*, \underline{q_2}^*, \overline{q_1}^*, \overline{q_1}^*) = \arg \max_{\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}} W(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}). \tag{7}$$

By Proposition 1, the quantities specified in the uninformed principal's contract, denoted $\{\underline{q_1}^{\mathrm{SB}}, \underline{q_2}^{\mathrm{SB}}, \overline{q_1}^{\mathrm{SB}}, \overline{q_1}^{\mathrm{SB}}\}$, are those that maximize $\Pi(\cdot)$:

$$\begin{aligned}(\underline{q_1}^{\mathrm{SB}}, \underline{q_2}^{\mathrm{SB}}, \overline{q_1}^{\mathrm{SB}}, \overline{q_1}^{\mathrm{SB}}) &= \arg \max_{\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}} \Pi(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}) \\ &= \arg \max_{\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}} W(\underline{q_1}, \underline{q_2}, \overline{q_1}, \overline{q_1}) - \nu \cdot \Phi(\underline{q_1}, \underline{q_2}). \end{aligned} \tag{8}$$

Observe two important properties of the two objective functions $W(\cdot)$ and $\Pi(\cdot)$. First, both objective functions are additively separable in $(\underline{q_1}, \underline{q_2})$ and $(\overline{q_1}, \overline{q_2})$. Hence, in both cases, the marginal returns to $\underline{q_1}$ and $\underline{q_2}$ are independent of $\overline{q_1}$ and $\overline{q_2}$ and vice versa. Consequently, the optimal values of $\overline{q_1}$ and $\overline{q_2}$ in both cases are independent of whether we condition on the remaining variables $\underline{q_1}$ and $\underline{q_2}$ or not, and vice versa for the optimal values of $\underline{q_1}$ and $\underline{q_2}$. Further, the two objective functions differ only in the additive term $\nu \cdot \Phi(\underline{q_1}, \underline{q_2})$, which is also independent of $\overline{q_1}$ and $\overline{q_2}$. From this, the following result should be obvious:

Proposition 2. *Under the uninformed principal, the efficient agent still produces efficient quantities in both periods, so* $\overline{q_1}^{\mathrm{SB}} = \overline{q_1}^*$ *and* $\overline{q_2}^{\mathrm{SB}} = \overline{q_2}^*$.

Proof. By differentiation of the objective functions, it is easily checked that the principal's and social returns to $\overline{q_1}$ and $\overline{q_2}$ are equal and, due to additive separability of both functions in $(\overline{q_1}, \overline{q_2})$ and $(\underline{q_1}, \underline{q_2})$, independent of the associated values of $\underline{q_1}$ and $\underline{q_2}$ in each case:

$$\begin{aligned}\frac{\partial}{\partial \overline{q_1}} W(\cdot) &= \frac{\partial}{\partial \overline{q_1}} \Pi(\cdot) = \nu \left[S'(\overline{q_1}) - c_1 - \delta \overline{q_2} \frac{\partial}{\partial \overline{q_1}} c_2(\overline{q_1}, \overline{\theta}) \right] \\ \frac{\partial}{\partial \overline{q_2}} W(\cdot) &= \frac{\partial}{\partial \overline{q_2}} \Pi(\cdot) = \nu \delta \left[S'(\overline{q_1} - c_2(\overline{q_1}, \overline{\theta}) \right]. \end{aligned}$$

---

so that concavity of the objective function is equivalent to

$$(1-\nu) \left\{ (1-\nu) S''(\underline{q_1}) - \delta \underline{q_2} \left[\frac{\partial^2}{\partial \underline{q_1}^2} c_2(\underline{q_1}, \underline{\theta}) - \nu \frac{\partial^2}{\partial \underline{q_1}^2} c_2(\underline{q_1}, \overline{\theta}) \right] \right\} S''(\underline{q_2}) \geq \delta \left\{ \frac{\partial}{\underline{q_1}} c_2(\underline{q_1}, \underline{\theta}) - \nu \frac{\partial}{\partial \underline{q_1}} c_2(\underline{q_1}, \overline{\theta}) \right\}^2.$$

Hence, any values of $(\overline{q_1}, \overline{q_2})$ which maximize $W(\cdot)$ also maximize $\Pi(\cdot)$, independently of the associated values of $(\underline{q_1}, \underline{q_2})$. The result then immediately follows by the fact that we have assumed the maximizer to be interior and unique by the assumed concavity of the objective. $\square$

This result is akin to the 'no distortion at the top' result, familiar from the static model, which states that incomplete information does not alter production levels of the efficient $\overline{\theta}$-agent. Intuitively, this result is due to the fact that in designing the optimal contract, the principal need worry only about 'downward lying', i.e. the efficient agent claiming to be inefficient. This agency cost, $\nu \cdot \Phi(\underline{q_1}, \underline{q_2})$, is only a function of the inefficient type's levels of output, $\underline{q_1}$ and $\underline{q_2}$, but unaffected by the efficient type's output schedule, so that there is no incentive to distort the good type's output schedule.

A more direct but perhaps less instructive way to see this result is to explicitly consider the first-order conditions on $\overline{q_1}^{\mathrm{SB}}$ and $\overline{q_2}^{\mathrm{SB}}$,

$$S'(\overline{q_1}^{\mathrm{SB}}) = c_1 + \delta \overline{q_2}^{\mathrm{SB}} \frac{\partial c_2(\overline{q_1}^{\mathrm{SB}}, \overline{\theta})}{\partial q_1}, \tag{9}$$

$$S'(\overline{q_2}^{\mathrm{SB}}) = c_2(\overline{q_1}^{\mathrm{SB}}, \overline{\theta}), \tag{10}$$

and noting that they are identical to the first-order conditions on first-best output $\overline{q_1}^*$ and $\overline{q_2}^*$ derived in Section 2, conditions (1) and (2).

However, the above optimization programs indicate that the inefficient type's output schedule, $\underline{q_1}$ and $\underline{q_2}$, will be subject to distortions due to the expected information rent $\nu \cdot \Phi(\underline{q_1}, \underline{q_2})$, which drives a wedge between social objectives and the uninformed principal's objectives. In a first step, we shall be interested in distortions in $\underline{q_1}$ and $\underline{q_2}$, respectively, *given* the values of all other variables. To this end, again with slight abuse of notation, define $\underline{q_2}^{\mathrm{SB}}(\underline{q_1})$ to be the level of second-period output $\underline{q_2}$ which maximizes the principal's objective function $\Pi(\cdot)$ given any first-period output $\underline{q_1}$ and the efficient type's output schedule $(\overline{q_1}, \overline{q_2})$ (recall that due to the additive separability, $\underline{q_1}^{\mathrm{SB}}(\cdot)$ will be independent of the latter two). Recall that in section 2, we have similarly defined conditional first-best output in period two, $q_2^*(\underline{q_1}|\underline{\theta}) \equiv \underline{q_2}^*(\underline{q_1})$, which we can now interpret as the value of $\underline{q_2}$ which maximizes expected social welfare $W(\cdot)$ given $\underline{q_1}$, $\overline{q_1}$, and $\overline{q_2}$. Then, the following result should be rather obvious:

Proposition 3. *For given first period output, private information about learning effects leads the uninformed principal to distort the bad type's second period output downward from its socially efficient level:* $\underline{q_2}^{\mathrm{SB}}(q_1) < \underline{q_2}^*(q_1)$ *for all* $q_1 \geq 0$.

Proof. This can be seen immediately by comparing the marginal social returns to $\underline{q_2}$ with the principal's returns to $\underline{q_2}$ given any level of $\underline{q_1}$:

$$\tfrac{\partial}{\partial \underline{q_2}} \Pi(\cdot) = \tfrac{\partial}{\partial \underline{q_2}} W(\cdot) - \nu \cdot \tfrac{\partial}{\partial \underline{q_2}} \phi(\cdot).$$

Since $\frac{\partial}{\partial \underline{q_2}}\phi > 0$ by the properties of $\Phi(\cdot)$ derived in Lemma 2, so that the informational rent is strictly increasing in $\underline{q_2}$, we known that $\frac{\partial}{\partial \underline{q_2}}\Pi < \frac{\partial}{\partial \underline{q_2}}W$, so that for any given first-period output $\underline{q_1}$, the principal's returns to marginally raising $\underline{q_2}$ fall short of the corresponding social returns. Given that the optima are unique and interior by assumption, the result then follows immediately from simple univariate robust comparative statics. $\square$

The reason that this result should come as no surprise is that for given first-period output levels, each type's second-period marginal costs c_2 are given. Hence, the principal's optimization problem is similar to the standard model of choosing single-period output with fixed type-dependent marginal costs, for which it is a familiar result that the less efficient type's output is distorted downward (the 'distortion at the bottom' result): The principal must trade off efficiency, as captured by joint surplus $W(\cdot)$, against expected rent payments to the efficient type, $\nu\Phi(\underline{q_1}, \underline{q_2})$. Clearly then, starting from the conditionally efficient level of second-period output which maximizes $W(\cdot)$, $\underline{q_2}^*$, marginally lowering $\underline{q_2}$ has no first-order efficiency effect, but decreases the informational rent by Lemma 2.

While the above approach of showing how the principal's and the social objective systematically differ with respect to the marginal effect of $\underline{q_2}$ aides intuition, note that given that uniqueness of the optimum and concavity of the objective are satisfied, we may again alternatively consider the first-order condition on second-period output in the principal's maximization problem,

$$\begin{aligned} S'(\underline{q_2}) &= c_2(\underline{q_1}, \underline{\theta}) + \frac{\nu}{1-\nu}\frac{\partial}{\partial q_2}\Phi(\underline{q_1}, \underline{q_2}) \\ &= c_2(\underline{q_1}, \underline{\theta}) + \frac{\nu}{1-\nu}\left[c_2(\underline{q_1}, \underline{\theta}) - c_2(\underline{q_1}, \overline{\theta})\right], \end{aligned} \tag{11}$$

and observe that the distortionary third term on the right-hand side will be positive.

Hence, for given first-period output, our model captures familiar effects. Let us now turn to distortionary incentives concerning first-period output, for now keeping second-period output fixed:

Proposition 4. *For given second period output, private information about the agent's learning effects leads the uninformed principal to distort the inefficient type's first period output*

- downward *if the efficient type learns faster than the inefficient type:* $\underline{q_1}^{\mathrm{SB}}(q_2) < \underline{q_1}^*(q_2)$ *for all* $q_2 > 0$*, and*
- upward *if the inefficient type learns faster than the efficient type:* $\underline{q_1}^{\mathrm{SB}}(q_2) > \underline{q_1}^*(q_2)$ *for all* $q_2 > 0$*.*

There is no distortion in first-period output if agents learn at the same rate, so $\underline{q_1}^{\mathrm{SB}}(q_2) = \underline{q_1}^*(q_2)$*.*

Proof. Again, consider the principal's marginal returns to increasing $\underline{q_1}$, which we may write as

$$\frac{\partial}{\partial \underline{q_1}}\Pi(\cdot) = \frac{\partial}{\partial \underline{q_1}}W(\cdot) - \nu \cdot \frac{\partial}{\partial \underline{q_1}}\Phi(\cdot).$$

Now, from Lemma 2, we know that the sign on $\frac{\partial}{\partial \underline{q_1}}\Phi$ depends on agents' relative learning rates. If the efficient type learns faster, so $\frac{\partial^2}{\partial q_1 \partial \theta}c_2 < 0$, then $\frac{\partial}{\partial \underline{q_1}}\Phi > 0$ by Lemma (2) whenever $\underline{q_2} > 0$, so that $\frac{\partial}{\partial \underline{q_1}}\Pi < \frac{\partial}{\partial \underline{q_1}}W$, i.e. for any second-period output $\underline{q_2} > 0$, the principal's returns to marginally raising $\underline{q_1}$ fall short of the corresponding social returns. On the contrary, if the inefficient agent learns faster, then $\frac{\partial}{\partial \underline{q_1}}\Pi > \frac{\partial}{\partial \underline{q_1}}W$ by analogous reasoning using Lemma 2, so that for any second-period output $\underline{q_2} > 0$, the principal's returns to marginally raising $\underline{q_1}$ exceed the corresponding social returns. Finally, if both agents learn at the same rate, then $\frac{\partial}{\partial \underline{q_1}}\Phi = 0$, so that $\frac{\partial}{\partial \underline{q_1}}\Pi = \frac{\partial}{\partial \underline{q_1}}W$. Again, given that the optima are unique and interior by assumption, each of the results then follow immediately from simple univariate robust comparative statics. □

To develop an intuition for this result (still keeping second period output fixed), recall that the difference in marginal costs between types is $c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})$. Loosely speaking, this difference captures the *extent of agents' private knowledge* in the second period: the larger the difference, the larger the principal's uncertainty about types' costs, and hence the larger the rent that must be given up to the good type in order to induce him not to lie about his type in period two. Now by choice of $\underline{q_1}$, the principal may influence the difference in period-two marginal costs and thereby the extent to which agents are privately informed. Whether this induces the principal to distort first-period output up- or downward then depends on whether the difference $c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})$ is increasing or decreasing in first period output, which in turn depends on the sign of $\partial^2 c_2 / \partial q_1 \partial \theta$, i.e. how agents' rates of learning differ. This allows us to distinguish the following cases (inspection of Figure 1 may help in keeping tabs on cases):

- Agents learn at the same rate: $\partial^2 c_2 / \partial q_1 \partial \theta = 0$.
  In this case, the difference in marginal costs between types, $c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})$, is *independent* of first period output. Hence, given any period-two output schedules, the principal has no incentive to distort first period output away from its efficient level, as this has no impact on agents' extent of private knowledge and thereby the information rent payable to the good type.

- Efficient type also learns faster: $\partial^2 c_2 / \partial q_1 \partial \theta < 0$.
  The difference in types' second period marginal costs, $c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})$, now increases in first period output: As the type which already enjoys lower second period costs learns even faster, higher first period output serves to widen cost

differences between types and thereby the extent of private information. To reduce the rent payable to the good type, the principal will therefore want to distort $\underline{q_1}$ downward.

- Inefficient type learns faster: $\partial^2 c_2/\partial q_1 \partial\theta > 0$.

  When the good type's rate of learning is lower than the bad type's, first period output must be raised in order to reduce $c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})$: As the bad type learns faster with increasing q_1, his marginal costs will approach the good type's. The principal will therefore want to distort $\underline{q_1}$ upward from its socially efficient level.

Once more, let us note for completeness that the above result may have been derived directly by considering the principal's first-order condition on first-period output,

$$
\begin{aligned}
S'(\underline{q_1}) &= c_1 + \delta \underline{q_2} \frac{\partial c_2(\underline{q_1}, \underline{\theta})}{\partial q_1} + \frac{\nu}{1-\nu} \frac{\partial \Phi(\underline{q_1}, \underline{q_2})}{\partial q_1} \\
&= c_1 + \delta \underline{q_2} \frac{\partial c_2(\underline{q_1}, \underline{\theta})}{\partial q_1} + \frac{\nu}{1-\nu} \delta \underline{q_2} \left[\frac{\partial c_2(\underline{q_1}, \underline{\theta})}{\partial q_1} - \frac{\partial c_2(\underline{q_1}, \overline{\theta})}{\partial q_1} \right], \qquad (12)
\end{aligned}
$$

and noting that the distortionary third term depends on which type of agent learns faster.

The results so far concerning the inefficient type's output schedule are of course only partial, as in each case we have held fixed the other period's output schedule. Ultimately, the principal will simultaneously distort both $\underline{q_1}$ and $\underline{q_2}$ relative to the full-information efficiency benchmark in order to implement the optimal trade-off between allocative efficiency, as represented by $W(\cdot)$, and expected rent payments to the good type, $\nu \cdot \Phi(\underline{q_1}, \underline{q_2})$. Put differently, the above analysis reveals the direction in which each periods' output under asymmetric information could be manipulated in order to increase social welfare while keeping the other periods' output fixed. This leaves open the direction of efficiency enhancing output changes when both periods' output can be manipulated simultaneously. Clearly then, the complementarity relation between $\underline{q_1}$ and $\underline{q_2}$ in the objective functions will be crucial.

To make this specific, consider the case in which the inefficient agent learns faster. We then know already that $\frac{\partial}{\partial \underline{q_1}} \Pi > \frac{\partial}{\partial \underline{q_1}} W$ and $\frac{\partial}{\partial \underline{q_2}} \Pi < \frac{\partial}{\partial \underline{q_2}} W$, so that starting from the efficient allocation, the principal has an upward disortionary incentive for first-period output and a downward distortionary incentive for second-period output. Now if we had $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W < 0$ and $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi < 0$, then these incentives would reinforce each other as lower second-period output $\underline{q_2}$ would further raise the incentive to distort first-period output $\underline{q_1}$ upward and vice versa, and simple supermodular analysis would confirm this intuition by showing that overall, the equilibrium will involve an upward distortion in first-period output and a downward distortion in second-period output. However, we have previously argued in Section 2 that outputs are complementary in the social objective, so $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W > 0$, as is easily checked. Furthermore, we know from Lemma 2 that $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Phi > 0$ when the inefficient agent learns faster, as in this case a higher volume of first-period output reduces differences in marginal costs, which in turn reduces agency costs per unit of second-period output and thereby increases the incentive to raise second-period output. Hence, $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi = \frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W + \frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Phi > \frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W > 0$, so that the complementarity in first- and second-period output schedules work against the partial first-order effects, which should make it difficult to derive robust results on overall distortions when the inefficient agent learns faster.

Next, consider the case in which the efficient agent learns faster. We then know that $\frac{\partial}{\partial \underline{q_1}} \Pi < \frac{\partial}{\partial \underline{q_1}} W$ and $\frac{\partial}{\partial \underline{q_2}} \Pi < \frac{\partial}{\partial \underline{q_2}} W$, so that starting from the efficient allocation, the principal has a downward disortionary incentive concerning both periods' outputs. Now in this case, $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W > 0$ and $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi > 0$ would cause these incentives to reinforce each other. We have already argued above that indeed, $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} W > 0$. However, when the efficient agent learns faster, then $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Phi < 0$ by Lemma 2: A higher level of first-period output $\underline{q_1}$ then further increases the marginal cost differences between types, increasing second-period agency costs per piece, and thereby giving an incentive to lower second-period output $\underline{q_2}$. In sum, it is easily seen that $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi$ will be positive for some, and negative for other settings, and only in the former case will supermodular analysis deliver a clear result.[11]

Nevertheless, we can derive the following general result concerning overall equilibrium distortions in the case of the efficient agent also learning faster:

Proposition 5. *If the more efficient agent also learns faster, then private information causes an overall downward distortion in both first- and second-period output for the inefficient type, so* ${\underline{q_1}}^{\mathrm{SB}} < {\underline{q_1}}^*$ *and* ${\underline{q_2}}^{\mathrm{SB}} < {\underline{q_2}}^*$.

Proof. To show this result, we consider each of the possible quadrants around $({\underline{q_1}}^*, {\underline{q_2}}^*)$ in turn. First, consider the case in which ${\underline{q_1}}^{\mathrm{SB}} \geq {\underline{q_1}}^*$ and ${\underline{q_2}}^{\mathrm{SB}} \geq {\underline{q_2}}^*$. We know that this cannot be optimal since the uninformed principal's objective function is concave by assumption and both partials are strictly negative at $({\underline{q_1}}^*, {\underline{q_2}}^*)$

[11] Indeed, it is easily checked that $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi > 0$ whenever

$$\nu \cdot \left| \frac{\partial}{\partial q_1} c_2(\underline{q_1}, \overline{\theta}) \right| < \left| \frac{\partial}{\partial q_1} c_2(\underline{q_1}, \underline{\theta}) \right|, \qquad \forall \underline{q_1} \geq 0,$$

so that either the efficient agent does not learn too much faster than the inefficient agent or efficient agents are relatively scarce, a condition which we will see again in Section 3.3's analysis of whether learning effects are exploited at all. If the inequality is reversed for all $\underline{q_1}$, then $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Pi < 0$.

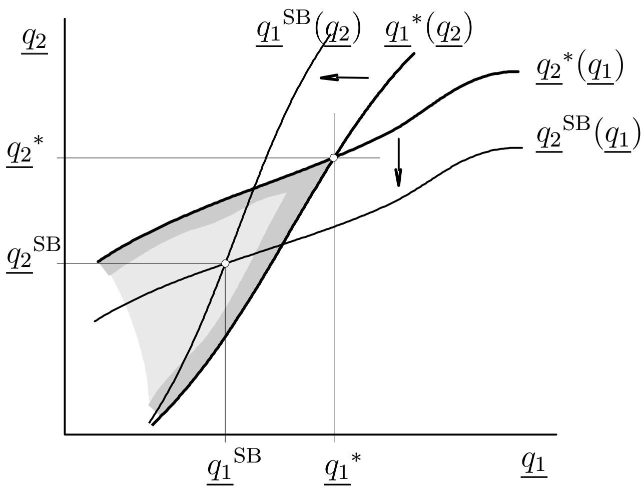


Figure 2: Restrictions on Equilibrium Quantities when the Efficient Agent also Learns Faster.

whenever the efficient agent also learns faster.

Next, consider the case in which $\underline{q}_1^{\mathrm{SB}} \leq \underline{q}_1^*$ and $\underline{q}_2^{\mathrm{SB}} \geq \underline{q}_2^*$. Recall that the inefficient type's conditional first-best function for second-period output, $\underline{q}_2^*(q_1)$, is strictly increasing in q_1. Thus, whenever $\underline{q}_1^{\mathrm{SB}} \leq \underline{q}_1^*$, we must have $\underline{q}_2^*(\underline{q}_1^{\mathrm{SB}}) \leq \underline{q}_2^*(\underline{q}_1^*)$. But from Proposition 3, we know that $\underline{q}_2^{\mathrm{SB}}(\underline{q}_1^{\mathrm{SB}}) < \underline{q}_2^*(\underline{q}_1^{\mathrm{SB}})$, and hence whenever $\underline{q}_1^{\mathrm{SB}} \leq \underline{q}_1^*$, then $\underline{q}_2^{\mathrm{SB}} \equiv \underline{q}_2^{\mathrm{SB}}(\underline{q}_1^{\mathrm{SB}}) < \underline{q}_2^*(\underline{q}_1^*) \equiv \underline{q}_2^*$. Thus, $\underline{q}_1^{\mathrm{SB}} \leq \underline{q}_1^*$ and $\underline{q}_2^{\mathrm{SB}} \geq \underline{q}_2^*$ can never be optimal.

Finally, consider the case in which $\underline{q}_1^{\mathrm{SB}} \geq \underline{q}_1^*$ and $\underline{q}_2^{\mathrm{SB}} \leq \underline{q}_2^*$. Recall that conditional first-best output in the first period, $\underline{q}_1^*(q_2)$ is also increasing in second-period output, so that $\underline{q}_1^*(\underline{q}_2^{\mathrm{SB}}) \leq \underline{q}_1^*(\underline{q}_2^*)$ whenever $\underline{q}_2^{\mathrm{SB}} \leq \underline{q}_2^*$. But Proposition 4 tells us that whenever the more efficient agent also learns faster, $\underline{q}_1^{\mathrm{SB}}(\underline{q}_2^{\mathrm{SB}}) < \underline{q}_1^*(\underline{q}_2^{\mathrm{SB}})$. Hence, if the efficient agent also learns faster, then $\underline{q}_1^{\mathrm{SB}} \geq \underline{q}_1^*$ and $\underline{q}_2^{\mathrm{SB}} \leq \underline{q}_2^*$ can never be optimal. □

This result has a simple graphical representation, shown in Figure 2. Here, the first-best output combination is represented by the intersection of the conditionally optimal output curves, $\underline{q}_1^*(\underline{q}_2)$ and $\underline{q}_2^*(\underline{q}_1)$, in $(\underline{q}_1, \underline{q}_2)$-space. We know from our previous analysis that due to the complementarity of first-and second-period output in the social objective function, both curves have a positive slope. Furthermore, it can easily be seen that the social objective being concave by assumption implies that the $\underline{q}_1^*(\underline{q}_2)$-curve intersects the $\underline{q}_2^*(\underline{q}_1)$-curve from below at $(\underline{q}_1^*, \underline{q}_2^*)$ in $(\underline{q}_1, \underline{q}_2)$-space, as shown in the figure.[12] Now Proposition 3 tells us that second-period output will be

[12]Formally, this can be shown by explicitly writing down the slopes of the two curves using their defining first-order conditions, (12) and (11), and using the explicit concavity requirement in

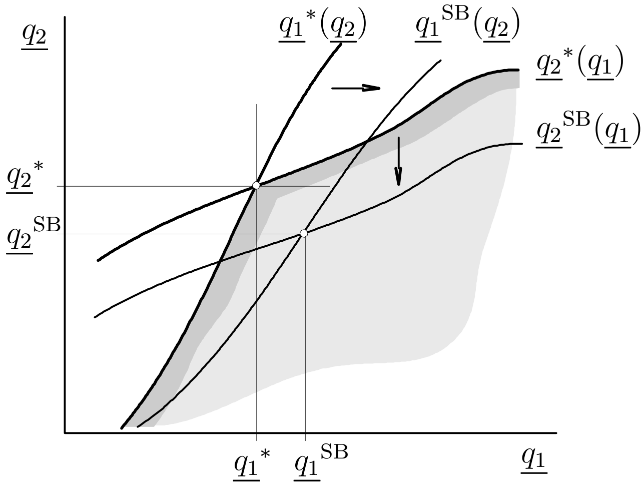


Figure 3: Restrictions on Equilibrium Quantities when the Inefficient Agent Learns Faster.

distorted downward given any first-period output level, so that the $\underline{q}_2{}^{\mathrm{SB}}(\underline{q}_1)$-curve will lie to the left of the $\underline{q}_2{}^*(\underline{q}_1)$-curve. Furthermore, Proposition 4 tells us that whenever the more efficient agent also learns faster, first-period output will be distorted downward given any second-period output level, so that the $\underline{q}_1{}^{\mathrm{SB}}(\underline{q}_2)$-curve will lie to the left of the $\underline{q}_1{}^*(\underline{q}_2)$-curve. But then the equilibrium under private information, which is determined by the intersection of the $\underline{q}_2{}^{\mathrm{SB}}(\underline{q}_1)$- and the $\underline{q}_1{}^{\mathrm{SB}}(\underline{q}_2)$-curve must lie to the left of the $\underline{q}_1{}^*(\underline{q}_2)$-curve and below the $\underline{q}_2{}^*(\underline{q}_1)$-curve, the shaded area in Figure 2. Due to the described properties of the two conditional first-best curves, this are will always lie strictly in the southeast quadrant, i.e. where $\underline{q}_1 < \underline{q}_1{}^*$ and $\underline{q}_2 < \underline{q}_2{}^*$.

Why this sort of argument no longer goes through when the less efficient agent learns faster should become clear from Figure 3. Again, the downward distortion in second-period output implies that the equilibrium output combination under private information lies below the $\underline{q}_2{}^*(\underline{q}_1)$-curve. However, Proposition 4 in this case tells us that there will be an upward distortion in first-period output given any second-period output, so that the new equilibrium must lie to the right of the $\underline{q}_1{}^*(\underline{q}_2)$-curve. Thus, when the less efficient agent learns faster, we can only preclude equilibria with $\underline{q}_1{}^{\mathrm{SB}} < \underline{q}_1{}^*$ and $\underline{q}_2{}^{\mathrm{SB}} > \underline{q}_2{}^*$.

While this by itself is of course no proof that all other three equilibria may indeed occur when the less efficient agent learns faster, it is easy to find parametric examples which indeed host all three equilibria for different parameter values. Particularly, we note the following result:

footnote 5.

Proposition 6. *If the less efficient agent learns faster, then private information can cause an overall upward distortion in both first- and second-period output for the inefficient type, so* $\underline{q_1}^{\mathrm{SB}} > \underline{q_1}^*$ *and* $\underline{q_2}^{\mathrm{SB}} > \underline{q_2}^*$.

Proof. To see this, consider the case in which the second period cost function is given by the linear form $c_2(q_1, \theta) = c(\theta) - \gamma(\theta)q_1$ with $c(\underline{\theta}) > c(\overline{\theta}) > 0$ and $\gamma(\theta) \geq 0$, and where the principal's objective function is given by the quadratic form $S(q_t) = aq_t - bq_t^2$, where $a, b > 0$. Note that the less efficient agent learning faster simply means that $\gamma(\underline{\theta}) > \gamma(\overline{\theta})$ in this linear specification. Then, for instance, setting $a = 100$, $b = 80$, $c_1 = 75$, $c(\underline{\theta}) = 50$, $c(\overline{\theta}) = 30$, $\gamma(\underline{\theta}) = 95$, $\gamma(\overline{\theta}) = 60$, $\delta = 0.7$, and $\nu = 0.5$ (the concavity requirement on the principal's objective function are easily checked for this parameterization), we obtain an optimal contract involving $\underline{q_1} = 0.49$ and $\underline{q_2} = 0.58$, whereas the first best would entail $\underline{q_1}^* = 0.38$ and $\underline{q_2}^* = 0.54$. □

3.3 Are Learning Effects Exploited At All?

Our analysis thus far has been focussed on comparing the output levels under private information to their constrained and unconstrained first-best counterparts under complete information. Additionally, however, we should be interested in knowing whether learning effects are exploited at all in the first place (for the bad type). For this comparison, the natural benchmark for first-period output is that which would result if either (i) first-period output in fact had no impact on second-period marginal costs, so $\frac{\partial}{\partial q_1}c_2 \equiv 0$, or (ii) both principal and agent behaved myopically, taking into account only first-period payoffs. In either hypothetical case, the optimal choice of q_1 will equalize marginal returns in period one, $S'(q_1)$, with marginal costs c_1. First period output will be higher than this whenever $S'(q_1) < c_1$, and lower whenever $S'(q_1) > c_1$. Motivated by this, we introduce the following terminology:

Definition 2 (Exploitation of Learning Effects). We shall say that learning effects are *exploited* whenever first-period marginal returns to output fall short of the marginal costs of output, so $S(q_1) < c_1$.

Recall from (12) that the first order condition on first-period output is

$$S'(\underline{q_1}) = c_1 + \delta \underline{q_2} \frac{\partial c_2(\underline{q_1}, \underline{\theta})}{\partial q_1} + \frac{\nu}{1-\nu} \delta \underline{q_2} \left[\frac{\partial c_2(\underline{q_1}, \underline{\theta})}{\partial q_1} - \frac{\partial c_2(\underline{q_1}, \overline{\theta})}{\partial q_1} \right].$$

If the right-hand side of this condition falls short of c_1, then first-period output of the optimal contract will exceed the benchmark output, so learning effects are indeed exploited even under private information. If the right-hand side is greater than c_1, the reverse is true. Thus, we may state the following result:

Proposition 7. *If the efficient agent does not learn too much faster than the inefficient agent, so*

$$\nu \cdot \left| \frac{\partial}{\partial q_1} c_2(\underline{q_1}, \overline{\theta}) \right| < \left| \frac{\partial}{\partial q_1} c_2(\underline{q_1}, \underline{\theta}) \right|, \qquad \forall \underline{q_1} \geq 0, \tag{13}$$

then learning effects are exploited in the sense that first-period marginal returns to output fall short of marginal costs c_1. If the good agent learns sufficiently faster than the bad agent, on the other hand (so that the inequality is reversed), then learning effects are not exploited, so first-period marginal returns to output exceed marginal costs.

It should be pointed out that describing condition (13) as 'the efficient agent not learning too much faster than the inefficient agent' implicitly assumes that the ratio of types, as given by ν, is fixed. More generally, (13) represents a joint condition on both the prevalence of types that will be met whenever the efficient agent does not learn too much faster or efficient types are scarce enough. Either constellation ensures that incentives to distort first-period output downward for rent-extraction purposes do not outweigh the gains achievable through learning effects: The efficient agent not learning too much faster implies that the informational rent payable to the efficient type—whenever increasing in $\underline{q_1}$—is not too large, whereas ν being low implies that this informational rent is less likely to have to be paid in the first place, thus decreasing the motive for rent-extraction relative to efficiency concerns. Finally, as a straightforward corollary, note that learning effects (for the inefficient type) will always be exploited if the efficient agent doesn't learn at all, so $\frac{\partial}{\partial q_1} c_2(q_1, \overline{\theta}) \equiv 0$.

4 Conclusion

Our model has analyzed how the introduction of privately known learning capabilities into the standard dynamic model of adverse selection influences incentive design. The main focus has been on whether private information leads to an under- or an over-exploitation of learning effects relative to the efficient level. First, we have seen that in the familiar 'no distortion at the top'-fashion, the more efficient agent (i.e., having lower second period costs given any first period output) will produce efficient levels of first- and second-period output. Concerning the less efficient agent, we have shown that whether first-period production is inefficiently high or low relative to second-period output crucially depends on whether the inefficient type learns faster (so that second-period marginal costs converge with rising first-period output), or whether the efficient type learns faster (causing second-period marginal costs to diverge). If second-period costs diverge, our results parallel those in Lewis and Yildirim (2002a) in that learning effects will be under-exploited. If second-period costs converge, however, our model predicts an over-exploitation of learning effects. Furthermore, we have seen the size of this distortion in either case to depend on (i) the rate of divergence or convergence, and (ii) the prevalence of inefficient types. Second, we have seen that only when second-period costs diverge can we show that these effects unambiguously lead to an overall downward distortion in both periods' output relative to the fully efficient benchmark, whereas the direction of the overall distortion remains ambiguous for converging costs. Particulary, we have also seen that converging costs may indeed lead to an overall upward distortion in both periods' output. Third, we have seen the divergence in costs being strong enough may even lead to learning effects not being exploited in the first place, which again contrast with Lewis and Yildirim's (2002a) result that learning effects will always be exploited, albeit to an inefficiently low extent.

More generally, our analysis has shown that in order to predict an under- or over-exploitation of learning effects in dynamic adverse selection settings, it is important to identify whether these learning effects serve to magnify or to diminish differences in efficiency between types. Concerning vertical procurement relationships, for instance, we may seek to categorize supplying industries along these lines according to their technology. For example, consider rather simple low-tech inputs produced in more traditional 'bread-and-butter' industries where there is little scope for large technological improvements. Even if there is originally some scope for improvements through learning by doing, we would eventually expect all agents to 'catch on to the trick' (some types sooner, some later), after which there is little scope for further improvement. Thus, we would expect learning effects to quickly subside and to equalize agents' productivity. In such industries, our model would predict learning effects to be over- rather than under-exploited. In contrast, consider suppliers of more high-tech products such as the computer chip industry. Here, we would expect significant scope for long-run improvements in production technologies. Further, we would expect inherently more creative suppliers to ever improve their lead on less efficient suppliers through accumulated learning effects. For such suppliers, our model predicts learning effects to be under- rather than over-exploited. Similar technological arguments may also let us differentiate strategies concerning the regulation of monopolistic suppliers.

However, one may also think of possible applications of our model outside the realm of pure procurement and regulation settings. Consider employment relations, for instance, where how hard an agent works today influences his future productivity on the job. If we expect hard work to make a less efficient worker catch up with the more efficient worker's productivity, we should expect the employer to ask agents to work inefficiently hard on the job. If, on the other hand, we expect harder work today to magnify productivity differences between workers (as might be the case on more creative jobs), we should expect the employer to relax workers' workload below the efficient level. Slightly more far-fetched fields of application may include the credit market problems in the style of Freixas and Laffont (1990), where the borrower is privately informed about the returns to his project. If the future productivity of capital in the project systematically depends on the size of the loan today, then our analysis would predict inefficiently high first-period loans if higher loans cause second-period productivity to converge and vice versa if productivity converges. Finally, the insights may be applied to models of discrimination in quantity and quality by a monopolist supplier (see Maskin and Riley, 1984; Mussa and Rosen, 1978) if we assume that consumers get used to or even addicted to a good, so that consuming more (or a higher quality) of the good today increases consumers' willingness to pay tomorrow. The learning speed criterion of our model would then pertain to whether customers with a higher willingness to pay for the good also get used to the good faster, or whether it is the customers who value the good less who get used to it at a faster rate.

Nevertheless, some of these stories may appear to be stretching the simple two-period framework of our model, which brings up the scope for further extensions: First, it should be interesting to extend the model's number of periods in order to obtain more refined dynamics. In such an extension, we may want to model producers' technology such that previous periods' cost savings endure and can be further improved upon by learning by doing in subsequent periods. As a preliminary step, however, it may be interesting already to make first period costs type-dependent in our two-period model. Recall that in order to focus only on the returns-side to learning by doing (and to focus on the principal's problem of keeping checks on *second period* informational asymmetries), we have assumed above that different agents have identical first-period marginal costs. Relaxing this assumption may prove interesting in its own right. Finally, we have already hinted at the fact that assuming full long-term commitment on behalf of the principal may be unrealistic in certain situations, not least because actual long-term procurement relations and particularly regulation are often observed to be governed by short-term contracts (see Lewis and Yildirim, 2002a, on the latter). Indeed, when full commitment is available to the principal, we have seen that by the revelation principle, the dynamic problem essentially reduces a static one from a contracting perspective, in that the agent truthfully reveals his type prior to the first period which already determines all future transactions. Thus, it should be interesting to see how extending our analysis to situations in which the principal can only imperfectly commit alters the results.

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A Appendix

A.1 Properties of the $\Phi(q_1, q_2)$-function

The first assertion is due to

$$\Phi(q_1, q_2) = \delta q_2[c_2(q_1, \underline{\theta}) - c_2(q_1, \overline{\theta})] = -\delta q_2 \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial}{\partial \theta} c_2(q_1, \theta) d\theta$$

and $\partial c_2/\partial \theta < 0$ by assumption. The second assertion concerning the sign of $\partial \Phi(q_1, q_2)/\partial q_2 = \Phi(q_1, q_2)$ follows immediately then from taking the derivative of the right hand side. For the third assertion, note that

$$\frac{\partial}{\partial q_1} \Phi(q_1, q_2) = \delta q_2[\frac{\partial}{\partial q_1} c_2(q_1, \underline{\theta}) - \frac{\partial}{\partial q_1} c_2(q_1, \overline{\theta})] = -\delta q_2 \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial^2}{\partial q_1 \partial \theta} c_2(q_1, \theta) d\theta.$$

Observation 4 regarding the sign of $\frac{\partial^2}{\partial \underline{q_1} \partial \underline{q_2}} \Phi$ again follows from taking partials on the right hand side.

Finally, for observation 5 note that

$$\begin{aligned}
\frac{\partial^2}{\partial q_1^2} \Phi &= \delta q_2 \left[\frac{\partial^2 c_2(q_1, \underline{\theta})}{\partial q_1^2} - \frac{\partial^2 c_2(q_1, \overline{\theta})}{\partial q_1^2} \right] \\
\frac{\partial^2}{\partial q_2^2} \Phi &= 0 \\
\frac{\partial^2}{\partial q_1 \partial q_2} \Phi &= \delta \left[\frac{\partial c_2(q_1, \underline{\theta})}{\partial q_1} - \frac{\partial c_2(q_1, \overline{\theta})}{\partial q_1} \right],
\end{aligned}$$

so that the condition $\frac{\partial^2}{\partial q_1^2} \Phi \cdot \frac{\partial^2}{\partial q_2^2} \Phi \geq \frac{\partial^2}{\partial q_1 \partial q_2} \Phi$, which is necessary and sufficient for convexity, will only be satisfied for $\frac{\partial^2}{\partial q_1 \partial q_2} \Phi = 0$.

A.2 Proof of Proposition 1

Proof of Part (a)

The incentive constraint of an agent of type $\overline{\theta}$ is given by

$$\begin{aligned}
\overline{U} &= \overline{z_1} - c_1 \overline{q_1} + \delta[\overline{z_2} - c_2(\overline{q_1}, \overline{\theta}) \overline{q_2}] \\
&\geq \underline{z_1} - c_1 \underline{q_1} + \delta[\underline{z_2} - c_2(\underline{q_1}, \overline{\theta}) \underline{q_2}] \\
&= \underline{U} + \delta \underline{q_2}[c_2(\underline{q_1}, \underline{\theta}) - c_2(\underline{q_1}, \overline{\theta})].
\end{aligned}$$

Here, the first line simply restates the definition of the $\overline{\theta}$-type's utility from accepting his type's contract. The second line gives the $\overline{\theta}$-type's utility from accepting the $\underline{\theta}$-type's contract, which may not yield him more utility for the contract to be incentive compatible. Finally, the third line rewrites the second line by using the definition of $\underline{U}$, the inefficient type's rent, to substitute out for that type's aggregate transfers $\underline{z_1} + \delta \underline{z_2}$.

The incentive constraint of a $\underline{\theta}$-agent is obtained similarly. Summing up, incentive compatibility requires that—stated in rent-output terms—contracts satisfy

$$\begin{aligned}
\overline{U} &\geq \underline{U} + \delta \underline{q_2}[c_2(\underline{q_1}, \underline{\theta}) - c_2(\underline{q_1}, \overline{\theta})] \\
\underline{U} &\geq \overline{U} - \delta \overline{q_2}[c_2(\overline{q_1}, \underline{\theta}) - c_2(\overline{q_1}, \overline{\theta})].
\end{aligned}$$

Using the Φ-function as defined in (3), these incentive constraints may be written more compactly as

$$\overline{U} \geq \underline{U} + \Phi(\underline{q_1}, \underline{q_2}) \tag{$\overline{\text{IC}}$}$$

$$\underline{U} \geq \overline{U} - \Phi(\overline{q_1}, \overline{q_2}). \tag{$\underline{\text{IC}}$}$$

Next, let us consider the principal's objective. Given any set of type dependent payments and production levels, the principal's expected payoff from the truth-revealing contract is given by

$$\begin{aligned} &\nu\big\{S(\overline{q_1}) - \overline{z_1} + \delta[S(\overline{q_2}) - \overline{z_2}]\big\} \\ &+ (1-\nu)\big\{S(\underline{q_1}) - \underline{z_1} + \delta[S(\underline{q_2}) - \underline{z_2}]\big\}. \end{aligned}$$

Using our definition of rents $U(\theta)$ to substitute out for aggregate payments $z_1(\theta) + \delta z_2(\theta)$, the principal's payoff may equivalently be written in terms of rents and production levels as

$$\begin{aligned} &\nu\big\{S(\overline{q_1}) - c_1\overline{q_1} + \delta[S(\overline{q_2}) - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\big\} \\ &+ (1-\nu)\big\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\big\} \\ &- \nu\overline{U} - (1-\nu)\underline{U}. \end{aligned}$$

The principal maximizes this payoff by choice of $\{\overline{q_1}, \overline{q_2}, \overline{U}\}$ and $\{\underline{q_1}, \underline{q_2}, \underline{U}\}$ subject to incentive and participation constraints ($\overline{\text{IC}}$) through ($\underline{\text{PC}}$), which we restate here for convenience:

$$\overline{U} \geq \underline{U} + \Phi(\underline{q_1}, \underline{q_2}) \tag{$\overline{\text{IC}}$}$$

$$\underline{U} \geq \overline{U} - \Phi(\overline{q_1}, \overline{q_2}) \tag{$\underline{\text{IC}}$}$$

$$\overline{U} \geq 0 \tag{$\overline{\text{PC}}$}$$

$$\underline{U} \geq 0. \tag{$\underline{\text{PC}}$}$$

For argument's sake, we may treat this simultaneous optimization as a sequential decision in which the principal first chooses an allocation $\{\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}\}$ and then picks rents offered to agents, $\{\overline{U}, \underline{U}\}$ (and thereby implicitly transfers).

First off, note that any allocation such that $\Phi(\overline{q_1}, \overline{q_2}) < \Phi(\underline{q_1}, \underline{q_2})$ cannot be implemented by means of *any* choice of rents. To see this, note that the two incentive constraints taken together imply $\Phi(\underline{q_1}, \underline{q_2}) \leq \overline{U} - \underline{U} \leq \Phi(\overline{q_1}, \overline{q_2})$. Thus, only allocations $\{\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}\}$ satisfying the *implementability condition* $\Phi(\overline{q_1}, \overline{q_2}) \geq \Phi(\underline{q_1}, \underline{q_2})$ given by inequality (5) of the proposition can be realized by a direct revelation mechanism in the first place. Following the literature, we shall call any menu of allocations satisfying (5) *implementable*.

Next, let us make the following standard observations on constraints ($\overline{\text{IC}}$) through ($\underline{\text{PC}}$) at the optimum, given any desired allocation $\{\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}\}$:

1. Constraint ($\underline{\text{PC}}$) is binding at the optimum.

2. Constraint ($\overline{\text{PC}}$) can be neglected.

3. Constraint ($\overline{\text{IC}}$) is binding at the optimum.

4. For any implementable allocation $\{\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}\}$ (i.e., satisfying (5)), constraint ($\underline{\text{IC}}$) can be neglected.

(Given the properties of the Φ-function derived in Lemma 2, the proofs are standard; see for instance Salanié, 1997, p. 22–23.) By substituting for $\overline{U}$ and $\underline{U}$, assertions 1 and 3 allow us to rewrite the objective function in terms of outputs only as

$$\begin{aligned} &\nu\{S(\overline{q_1}) - c_1\overline{q_1} + \delta[S(\overline{q_2}) - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\} \\ &+ (1-\nu)\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\} \\ &- \nu\Phi(\underline{q_1}, \underline{q_2}). \end{aligned} \tag{4}$$

Next, assertion 2 lets us drop constraint ($\overline{\text{PC}}$). Finally, assertion 4 lets us replace constraint ($\underline{\text{IC}}$) with the implementability condition (5). Hence, the principal's mechanism design problem ultimately reduces to choosing $\{\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}\}$ so as to maximize (4) subject to (5). We have thus recast the principal's problem of offering a menu optimal incentive compatible contracts $\{z_1(\theta), z_2(\theta), q_1(\theta), q_2(\theta)\}$ into a reduced-form problem in which the principal decides on a menu of allocations $\{q_1(\theta), q_2(\theta)\}$, where any such menu is efficiently implemented by setting types' rents at $\overline{U} = \Phi(\underline{q_1}, \underline{q_2})$ and $\underline{U} = 0$, which in turn implicity determines discounted total transfers $z_1(\theta) + \delta z_2(\theta)$ for every menu item.

Proof of Part (b)

Recall that we have shown that the principal's problem of finding the optimal incentive compatible contract can be reduced to choosing a menu of allocations maximizing (4) subject to (5). What remains to be shown is that menus of allocations which violate (5) are never optimal, which we prove by showing that any allocation violating (5) can be replaced by another allocation which yields a strictly higher payoff to the principal and meets (5).[13]

Consider a menu of contracts with the $\underline{\theta}$-type being asked to produce $(\underline{q_1^\circ}, \underline{q_2^\circ})$ and the $\overline{\theta}$-type being asked to produce $(\overline{q_1^\circ}, \overline{q_2^\circ})$. Assume further that $\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) > \Phi(\overline{q_1^\circ}, \overline{q_2^\circ})$, i.e. that the menu of contracts violates the implementability condition (5).

Now consider the following set of allocations, (q_1, q_2), parameterized by $\alpha \in [0, 1]$, which constitute linear combinations of the original allocations $(\underline{q_1^\circ}, \underline{q_2^\circ})$ and $(\overline{q_1^\circ}, \overline{q_2^\circ})$:

$$\begin{bmatrix} q_1(\alpha) \\ q_2(\alpha) \end{bmatrix} = \alpha \begin{bmatrix} \underline{q_1^\circ} \\ \underline{q_2^\circ} \end{bmatrix} + (1-\alpha) \begin{bmatrix} \overline{q_1^\circ} \\ \overline{q_2^\circ} \end{bmatrix}, \qquad \text{where } \alpha \in [0, 1].$$

Now assume that in a new menu of contracts, the $\overline{\theta}$-type is asked to produce such a linear combination with $\alpha = \overline{\alpha}$, and the $\underline{\theta}$-type one with $\alpha = \underline{\alpha}$. Observe that the original

[13] Remember that we have shown that the principal optimally implements any menu of allocations $[(\underline{q_1}, \underline{q_2}); (\overline{q_1}, , \overline{q_2})]$ by setting transfers such that $\underline{U} = 0$ and $\overline{U} = \Phi(\underline{q_1}, \underline{q_2})$, which allows us to represent the principal's choice in reduced form as one of offering a menu of allocations only.

set of contracts results for $\underline{\alpha} = 1$ and $\overline{\alpha} = 0$. Assume further that $\overline{\alpha}$ and $\underline{\alpha}$ are chosen such that

$$\nu\overline{\alpha} = (1-\nu)(1-\underline{\alpha}). \tag{14}$$

The motivation for imposing (14) will become clear below. For now, note that this restriction on $\overline{\alpha}$ and $\underline{\alpha}$ ensures that expected first- and second-period quantities from the new contract (where expectations are taken over possible types θ) are the same as those resulting from the original contract. Next, let

$$\begin{aligned} g(\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}) \equiv & \quad \nu\big\{S(\overline{q_1} - c_1\overline{q_1} + \delta[S(\overline{q_2} - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\big\} \\ & + (1-\nu)\big\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\big\} - \nu\Phi(\underline{q_1}, \underline{q_2}). \end{aligned}$$

denote the principal's objective function as given in (4). Since g is concave in its arguments by assumption,

$$\begin{aligned} g[q_1(\overline{\alpha}), q_2(\overline{\alpha}), q_1(\underline{\alpha}), q_2(\underline{\alpha})] \geq & \quad \overline{\alpha}\underline{\alpha}g[\underline{q_1^\circ}, \underline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}] + \quad \overline{\alpha}(1-\underline{\alpha})g[\underline{q_1^\circ}, \underline{q_2^\circ}, \overline{q_1^\circ}, \overline{q_2^\circ}] \\ & + (1-\overline{\alpha})\underline{\alpha}g[\overline{q_1^\circ}, \overline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}] + (1-\overline{\alpha})(1-\underline{\alpha})g[\overline{q_1^\circ}, \overline{q_2^\circ}, \overline{q_1^\circ}, \overline{q_2^\circ}] \end{aligned} \tag{15}$$

by Jensen's inequality. Next, we make use of the additive separability of $g(\cdot)$ in $(\underline{q_1}, \underline{q_2})$ and $(\overline{q_1}, \overline{q_2})$. Specifically, let

$$\begin{aligned} \overline{g}(\overline{q_1}, \overline{q_2}) &\equiv \big\{S(\overline{q_1}) - c_1\overline{q_1} + \delta[S(\overline{q_2}) - c_2(\overline{q_1}, \overline{\theta})\overline{q_2}]\big\} \\ \underline{g}(\underline{q_1}, \underline{q_2}) &\equiv \big\{S(\underline{q_1}) - c_1\underline{q_1} + \delta[S(\underline{q_2}) - c_2(\underline{q_1}, \underline{\theta})\underline{q_2}]\big\} - \frac{\nu}{1-\nu}\Phi(\underline{q_1}, \underline{q_2}), \end{aligned}$$

so that we may write the principal's payoff function as

$$g(\overline{q_1}, \overline{q_2}, \underline{q_1}, \underline{q_2}) = \nu\overline{g}(\overline{q_1}, \overline{q_2}) + (1-\nu)\underline{g}(\underline{q_1}, \underline{q_2}).$$

Then we may rewrite the right-hand side of inequality (15) as

$$\begin{aligned} & \overline{\alpha}\underline{\alpha}g[\underline{q_1^\circ}, \underline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}] + \quad \overline{\alpha}(1-\underline{\alpha})g[\underline{q_1^\circ}, \underline{q_2^\circ}, \overline{q_1^\circ}, \overline{q_2^\circ}] \\ & + (1-\overline{\alpha})\underline{\alpha}g[\overline{q_1^\circ}, \overline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}] + (1-\overline{\alpha})(1-\underline{\alpha})g[\overline{q_1^\circ}, \overline{q_2^\circ}, \overline{q_1^\circ}, \overline{q_2^\circ}] \\ & \quad = \overline{\alpha}\nu\overline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + (1-\overline{\alpha})\nu\overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) + \underline{\alpha}(1-\nu)\underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + (1-\underline{\alpha})(1-\nu)\underline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) \\ & \quad = \nu\overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) + (1-\nu)\underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + \overline{\alpha}\nu\left[\overline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) - \overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) + \underline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) - \underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ})\right] \\ & \quad = g(\overline{q_1^\circ}, \overline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}) + \overline{\alpha}\nu\left[\overline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) - \underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + \underline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) - \overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ})\right], \end{aligned}$$

where we get to the third line by using additive separability (i.e. by plugging in the additive decomposition of $g(\cdot)$ into $\overline{g}(\cdot)$ and $\underline{g}(\cdot)$ and collecting terms), from the third to the fourth line by using (14) to substitute out for $\underline{\alpha}$ and rearranging, and to the last line by collecting the first two terms.[14] Hence, from inequality (15), we have a lower bound on the extent to

[14] To aide interpretation, it may be helpful to note that the second line reveals that this payoff is equivalent to what would be obtained by offering a stochastic menu of contracts in which the good $\overline{\theta}$-type produces $(\underline{q_1^\circ}, \underline{q_2^\circ})$ with probability $\overline{\alpha}$ and $(\overline{q_1^\circ}, \overline{q_2^\circ})$ with probability $1-\overline{\alpha}$, and the bad $\underline{\theta}$-type produces $(\underline{q_1^\circ}, \underline{q_2^\circ})$ with probability $\underline{\alpha}$ and $(\overline{q_1^\circ}, \overline{q_2^\circ})$ with probability $1-\underline{\alpha}$.

which the payoff from the newly constructed contract exceeds that from the original one:

$$g[q_1(\overline{\alpha}), q_2(\overline{\alpha}), q_1(\underline{\alpha}), q_2(\underline{\alpha})] - g(\overline{q_1^\circ}, \overline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}) \\ \geq \overline{\alpha}\nu \left[\overline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) - \underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + \underline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) - \overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) \right].$$

By plugging back in the definitions of $\overline{g}(\cdot)$ and $\underline{g}(\cdot)$, we may rewrite the right-hand term in brackets as

$$\begin{aligned} &\overline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) - \underline{g}(\underline{q_1^\circ}, \underline{q_2^\circ}) + \underline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) - \overline{g}(\overline{q_1^\circ}, \overline{q_2^\circ}) \\ &\quad = -\delta c_2(\underline{q_1^\circ}, \overline{\theta})\underline{q_2^\circ} + \delta c_2(\underline{q_1^\circ}, \underline{\theta})\underline{q_2^\circ} + \frac{\nu}{1-\nu}\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) \\ &\quad\quad - \delta c_2(\overline{q_1^\circ}, \underline{\theta})\overline{q_2^\circ} - \frac{\nu}{1-\nu}\Phi(\overline{q_1^\circ}, \overline{q_2^\circ}) + \delta c_2(\overline{q_1^\circ}, \overline{\theta})\overline{q_2^\circ} \\ &\quad = \Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) + \frac{\nu}{1-\nu}\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) - \Phi(\overline{q_1^\circ}, \overline{q_2^\circ}) - \frac{\nu}{1-\nu}\Phi(\overline{q_1^\circ}, \overline{q_2^\circ}) \\ &\quad = \frac{\nu}{1-\nu}\left[\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) - \Phi(\overline{q_1^\circ}, \overline{q_2^\circ})\right], \end{aligned}$$

so that the lower bound may be rewritten as

$$g[q_1(\overline{\alpha}), q_2(\overline{\alpha}), q_1(\underline{\alpha}), q_2(\underline{\alpha})] - g(\overline{q_1^\circ}, \overline{q_2^\circ}, \underline{q_1^\circ}, \underline{q_2^\circ}) \geq \overline{\alpha}\frac{\nu^2}{1-\nu}\left[\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) - \Phi(\overline{q_1^\circ}, \overline{q_2^\circ})\right].$$

Now since $\Phi(\underline{q_1^\circ}, \underline{q_2^\circ}) - \Phi(\overline{q_1^\circ}, \overline{q_2^\circ}) > 0$ by assumption, the right-hand-side expression is strictly positive for any $\overline{\alpha} > 0$. Thus, we have shown that any new set of contracts with $\overline{\alpha} > 0$ yields a strictly higher payoff to the principal than the original set of contracts.

Finally, we want to argue that while the original contract violates the implementability condition (5), we can pick $\overline{\alpha} > 0$ such that the resulting new contract meets (5). To this end, for any α, define $\widetilde{\Phi}(\alpha) \equiv \Phi[q_1(\alpha), q_2(\alpha)]$. The implementability condition (5) for the new set of contracts parameterized by $\overline{\alpha}$ and $\underline{\alpha}$ can then compactly be written as

$$\widetilde{\Phi}(\overline{\alpha}) > \widetilde{\Phi}(\underline{\alpha}).$$

Observe first that $\widetilde{\Phi}$ is monotonous in α. To see this, note that

$$\tfrac{\partial}{\partial\alpha}\widetilde{\Phi} = \tfrac{\partial}{\partial q_1}\Phi \cdot (\underline{q_1^\circ} - \overline{q_1^\circ}) + (\underline{q_2^\circ} - \overline{q_2^\circ}) \cdot \tfrac{\partial}{\partial q_2}\Phi,$$

where the signs on $\partial\Phi/\partial q_1$ and $\partial\Phi/\partial q_2$ are constant by Lemma 2. Furthermore, since $\Phi(\overline{q_1^\circ}, \overline{q_2^\circ}) < \Phi(\underline{q_1^\circ}, \underline{q_2^\circ})$ by assumption, where the former results for $\alpha = 0$ and the latter for $\alpha = 1$, $\widetilde{\Phi}$ must be strictly increasing in α.

Depending on whether there are more or less good than bad types (i.e. whether $\nu \geq 1/2$ or $\nu \leq 1/2$), we can now set $\overline{\alpha}$ and $\underline{\alpha}$ as follows:

1. Assume that bad types are more prevalent, so that $\nu \leq 1/2$. Then, we can set $\overline{\alpha} = 1$ and, by (14), $\underline{\alpha} = (1-2\nu)/(1-\nu) \in [0, 1)$. Then $\widetilde{\Phi}(\overline{\alpha}) = \Phi(\underline{q_1^\circ}, \underline{q_2^\circ})$. Furthermore, since $\widetilde{\Phi}(1) = \Phi(\underline{q_1^\circ}, \underline{q_2^\circ})$ and we know both that $\widetilde{\Phi}$ is strictly increasing in α and that $\underline{\alpha} < 1$, we must have $\widetilde{\Phi}(\underline{\alpha}) < \Phi(\underline{q_1^\circ}, \underline{q_2^\circ})$. Thus, the new contract satisfies (5).

2. When good types are more prevalent, so $\nu \geq 1/2$, we can set $\overline{\alpha} = (1-\nu)/\nu \in (0,1]$ and $\underline{\alpha} = 0$. Now $\widetilde{\Phi}(\underline{\alpha}) = \Phi(\overline{q_1^\circ}, \overline{q_2^\circ})$ and $\widetilde{\Phi}(\overline{\alpha}) > \Phi(\overline{q_1^\circ}, \overline{q_2^\circ})$, where the latter again follows from $\widetilde{\Phi}(0) = \Phi(\overline{q_1}, \overline{q_2})$, $\overline{\alpha} > 0$, and from $\widetilde{\Phi}$ being strictly increasing in α.

In sum, we have thus shown that for any contract involving an allocation that violates the implementability condition (5), we can find a new incentive compatible contract which (i) yields a strictly higher payoff the the principal and (ii) meets the implementability condition. Hence, the implementability condition can be neglected in the principal's optimization problem.

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